PHYSICS

SECTION-1

1.(C) $\omega = \theta^2 + 2\theta$

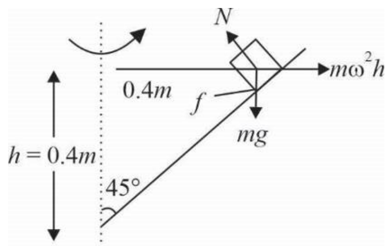
$\alpha = \dfrac{\omega d\omega}{d\theta} = (\theta^2 + 2\theta)(2\theta + 2)$

At $\theta = 1 rad$.

$\omega = 3 rad/s$ and $\alpha = 12 rad/s^2$

$a_t = \alpha R = 12 m/s^2$; $a_r = \omega^2 R = 9 m/s^2$; $A_T = \sqrt{a_t^2 + a_r^2} = 15 m/s^2$

2.(B) $N = \dfrac{mg}{\sqrt{2}} + \dfrac{m\omega^2 h}{\sqrt{2}}$

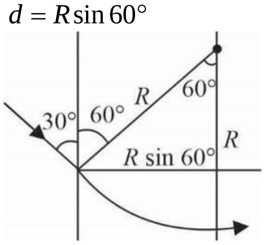


$\dfrac{m\omega^2 h}{\sqrt{2}} = \dfrac{mg}{\sqrt{2}} + f_L$

$m\omega^2 \dfrac{h}{\sqrt{2}} = \dfrac{mg}{\sqrt{2}} + \mu\left(\dfrac{mg}{\sqrt{2}} + \dfrac{m\omega^2 h}{\sqrt{2}}\right)$

$\omega = \sqrt{\dfrac{g}{h}\left(\dfrac{1+\mu}{1-\mu}\right)} = 10 rad/s$

3.(A) $d = R\sin 60°$

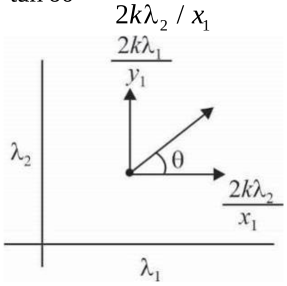


$\Rightarrow d = \dfrac{mv}{qB}\dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{3}v}{2aB}$

4.(B) $v = \sqrt{\dfrac{GM}{r}}$ (orbital speed)

$\Rightarrow GM = v^2 r \Rightarrow \dfrac{GM}{R^2} = \dfrac{v^2 r}{R^2} \Rightarrow g = \dfrac{v^2 r}{R^2}$

5.(D) $\tan 60° = \dfrac{2k\lambda_1 / y_1}{2k\lambda_2 / x_1}$



$\dfrac{\sqrt{3}\lambda_2}{x_1} = \dfrac{\lambda_1}{y_1}$; $\sqrt{3}\lambda_2 y_1 = \lambda_1 x_1$

6.(C) Vibrational energy of a non-rigid gas molecule is K_BT so, total energy $=\frac{5}{2}K_BT+K_BT=\frac{7}{2}K_BT$

$\therefore \quad C_v=\frac{7}{2}R$

7.(A) $\vec{v}_1=2\hat{i} \qquad \vec{F}_1=-2\hat{j}$

$\vec{v}_2=2\hat{j} \qquad \vec{F}_2=-2\hat{i}$

$\Rightarrow \quad \vec{B}$ is along $-\hat{k}$ $\qquad$ Hence $\vec{v}_3=2\hat{k} \;\Rightarrow\; \vec{F}=0$

8.(C) $L=I\omega$

$$I=\frac{mL^2}{3}+\frac{mL^2}{3}+\left[\frac{m\left(\sqrt{2}L\right)^2}{12}+m\left(\frac{L}{\sqrt{2}}\right)^2\right]=mL^2\left[\frac{2}{3}+\frac{1}{2}+\frac{1}{6}\right]=\frac{4mL^2}{3}\;;\; L=\frac{4}{3}mL^2\omega$$

9.(A)

10.(D) $U=kr^2 \;\Rightarrow\; F=-\frac{dU}{dr}=-2kr \;;\; 2kr=\frac{mv^2}{r} \;\Rightarrow\; v=\sqrt{\frac{2k}{m}} \;\; or \;\; T=\frac{2\pi r}{v}=2\pi\sqrt{\frac{m}{2k}}$

11.(C) $\Delta W=$ area under $P-t$ graph

$=\frac{1}{2}(4+6)\times 7=35J$

Work done = change in KE $\quad\Rightarrow\quad 35=\frac{1}{2}\times 2\times v^2-\frac{1}{2}\times 2\times(1)^2 \Rightarrow v=6m/s$

12.(A) at $t=t_1, \quad x_1=0 \quad\Rightarrow\quad 0=a\sin(\omega t_1+\pi/6)$

$\Rightarrow \quad \omega t_1+\pi/6=\pi$ $\qquad$ …..(i)

at $t=t_2, \quad x_2=0 \;\Rightarrow 0=a\sin(\omega t_2+\pi/4)$

$\Rightarrow \quad \omega t_2+\pi/4=\pi$ $\qquad$ ….(ii)

From (i) and (ii)

$\omega(t_1-t_2)=\frac{\pi}{4}-\frac{\pi}{6} \quad\Rightarrow\quad t_1-t_2=\frac{\pi}{12\omega}$

13.(A) Zero error $=0.0+4\times 0.01=0.04cm$

Reading $=2.5a+7\times 0.01=2.57cm$

Correct reading $=2.57cm-0.04cm=2.53cm$

14.(B) $X_L=X_R \Rightarrow \omega L=R \;\Rightarrow\; 2\times 3.14\times 50\times L=20 \;\Rightarrow\; L=63.7mH$

15.(D) $\vec{v}_1=3t\;\hat{i} \;;\; \vec{v}_2=24\cos 60°\hat{i}+24\sin 60°\hat{j}=12\hat{i}+12\sqrt{3}\hat{j}$

$\vec{v}_{21}=\vec{v}_2-\vec{v}_1=(12-3t)\hat{i}+12\sqrt{3}\hat{j}$

It is minimum when $12-3t=0 \Rightarrow t=4\sec$

16.(A) $\frac{K(4A)}{l}(100-\theta)=\frac{KA}{l}(\theta-50)$

$\Rightarrow \quad 400-4\theta=\theta-50 \Rightarrow 5\theta=450 \quad\Rightarrow\quad \theta=90°C$

17.(C) $f_1=f_0\left(\frac{330}{330-V}\right) \qquad f_2=f_0\left(\frac{330}{330+V}\right)$

$\frac{f_1-f_2}{f_0}\times 100=2\% \quad\Rightarrow\quad \frac{330}{330-V}-\frac{330}{330+V}=0.02$

$\Rightarrow\; 330\left[\frac{2V}{(330)^2-V^2}\right]=0.02 \;\Rightarrow\; 330V=0.01\times(330)^2 \;\Rightarrow\; V=3.3m/s$ (approx.)

18.(B) Energy of products $=939+940=1879$

$\Delta E+E_{reactions}-E_{productor}=1877-1879=-2MeV.$

Hence it must capture a $\gamma-$ ray photon of energy $2MeV$.

19.(C) $\vec{s}=-\hat{j}+\hat{k}$ $\quad\therefore\quad$ $\hat{s}=\dfrac{-\hat{j}+\hat{k}}{\sqrt{2}}$

20.(B) Velocity of bird $=v$

Velocity w.r.t. observer in water $=\dfrac{4}{3}v$; $12=4+\dfrac{4}{3}v \Rightarrow v=6\,cm/s$

SECTION-2

21.(1400) $P=\rho\dfrac{RT}{M_0} \Rightarrow \rho T=\text{constant}$

$$\Rightarrow \quad \frac{12}{4\times10^{-3}\times10^{6}}\times280=6\times10^{-4}\times T \quad\Rightarrow\quad T=1400K$$

22.(500) $\beta=\dfrac{\lambda D}{d}\Rightarrow\Delta\beta=\dfrac{\lambda}{d}\Delta D \quad\Rightarrow\quad 4\times10^{-5}=\dfrac{\lambda}{10^{-3}}\times8\times10^{-2}\Rightarrow\lambda=5\times10^{-7}m=500nm$

23.(8) $Q=a\sqrt{2gh}$

$$h=\frac{Q^2}{a^2\times2g}=\frac{\left(10^{-4}\right)^2}{10^{-8}\times2\times10}=5\ cm$$

Height above the ground $=3cm+5cm=8cm$

24.(250) $r=r_0\left(1+\alpha\Delta T\right)\Rightarrow r-r_0=r_0\alpha\Delta T$

$\Rightarrow 3\times10^{-3}=1\times1.2\times10^{-5}\Delta T \Rightarrow \Delta T=250°C$

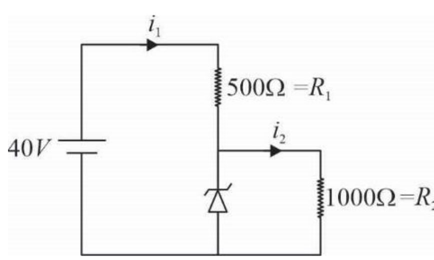
25.(5) $V_{common}=\dfrac{C_1V_1+C_2V_2}{C_1+C_2} \Rightarrow 2=\dfrac{(3\mu F)\times12}{(3\mu F)+(3\mu F)\times K}\Rightarrow2=\dfrac{12}{K+1} \Rightarrow K=5$

26.(1.20) $\Delta v=2f_0\left(\Delta l_2+\Delta l_1\right)=2\times300(0.1+0.1)\times10^{-2}=1.2m/s$

27.(15) at $t=0$, $I=0$; $E=\dfrac{LdI}{dt}\Rightarrow\dfrac{dI}{dt}=\dfrac{E}{L}=\dfrac{1.5}{0.1}=15A/s$

28.(15) $\vec{I}=\vec{P}_t-\vec{P}_i=3\times4\hat{j}-3\times3\hat{i}=12\hat{j}-9\hat{i}$; $\left|\vec{I}\right|=\sqrt{12^2+9^2}=15kgm/s$

29.(20) Potential difference across R_2



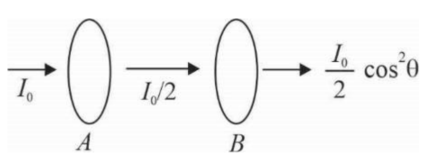
$\dfrac{R_2}{R_1+R_2}\times V$ is greater than zenor voltage

$\Rightarrow \quad i_2=\dfrac{V_z}{R_2}=\dfrac{20}{1000}A=20mA$

Current through R_1, $\quad i_1=\dfrac{40-V_Z}{R_1}=\dfrac{20}{500}=40mA$

Current through zeron diode $=i_1-i_2=20mA$

30.(53) $\dfrac{I_0}{2}\cos^2\theta=\dfrac{18}{100}I_0$



$\Rightarrow \cos^2\theta=\dfrac{9}{25}$

$\Rightarrow \cos\theta=\dfrac{3}{5} \Rightarrow \theta=53°$

CHEMISTRY

SECTION-1

1.(C) BOD value less than 5ppm is considered as clean water and BOD value more than 10ppm is considered as polluted water.

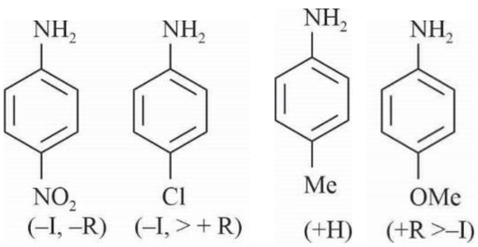
2.(C) $\because$ Solvent is H_2O, which is in excess

So using $m(\text{molality}) = \dfrac{x_2 \times 1000}{x_1 \times (Mo)_1}$

$\because \quad x_1 = 0.74 \qquad (Mol_1 = 18g)$

$x_2 = 1 - 0.74 = 0.26 \qquad \therefore \quad m = \dfrac{0.26 \times 1000}{0.74 \times 18} = 19.5$

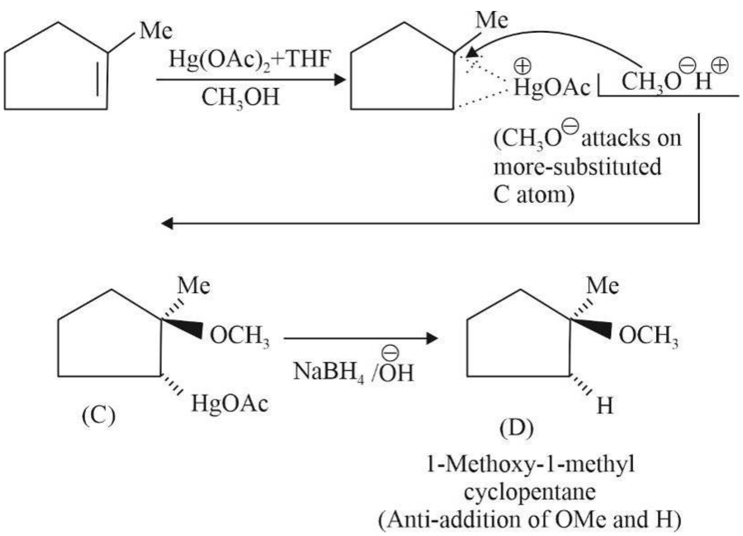
3.(C)



and basic strength $\propto \dfrac{1}{pK_b} \propto K_b \propto +R \propto \dfrac{1}{-R} \propto +I \propto \dfrac{1}{-I}$

So basic strength order $\Rightarrow$ IV > III > II > I, so pK_b order is IV < III < II < I

4.(A)



5.(D) All have same molarity, $pH \propto \dfrac{1}{\text{Acidic strength}} \propto$ Basic strength

$H_2SO_4 \rightarrow$ Acidic

NH_4Cl (Salt of weak base and strong acid)

Acidic but less than H_2SO_4.

$NaCl \Rightarrow$ (Neutral solution)

$NaOH \Rightarrow$ Basic (Strong Base)

So $\quad NaOH > NaCl > NH_4Cl > H_2SO_4$

6.(D) Thermal stability of compound is directly proportional to large anion of s-block elements:

$$\text{T.S} \propto \left(\frac{1}{\text{PP of cation}}\right)$$

And p.p. $\propto$ charge

$$\propto \left(\frac{1}{\text{size}}\right) \qquad \text{So,} \qquad \left[\text{T.S.} \propto \text{size} \propto \left(\frac{1}{\text{charge}}\right)\right]$$

So, $Sr[NO_3]_2$ is highly stable

But $Mg(NO_3)_2 \Rightarrow$ poor

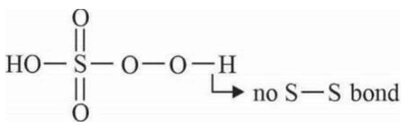
7.(C) From Reaction

$$\Delta n_g = 2 - 1 = 1$$

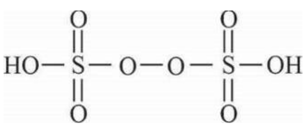
$$Kp = Kc(RT)^{\Delta n_g}$$

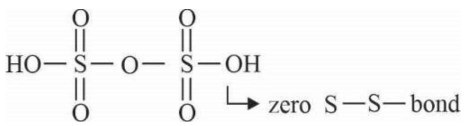
$$Kp = Kc(RT)^1$$

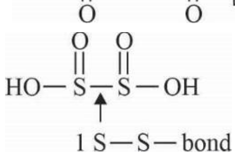
$$Kc = Kp(RT)^{-1}$$

8.(B) **(A)** $H_2SO_5 \Rightarrow$ 

(B) Marshalls acid is $(H_2S_2O_8)$

 $\Rightarrow$ zero S–S– bond

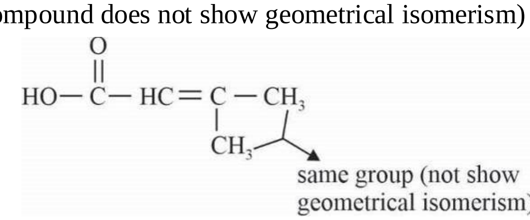
(C) $H_2S_2O_7 \Rightarrow$ 

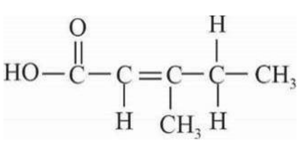
(D) $H_2S_2O_4 \Rightarrow$ 

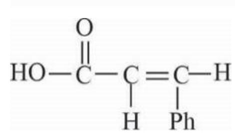
9.(C) Gutta percha is a synthetic rubber and its monomer is isoprene .
Since isoprene has EDG, so it is prepared by cationic addition polymerization

10.(B) Two same group on any C-atom of double bond $\Rightarrow$ (compound does not show geometrical isomerism)

(A) $H_3C-HC=CH-CH_3$

(B) 

(C) 

(D) 

11.(D) From $\Delta G° = \Delta H° - T\Delta S°$

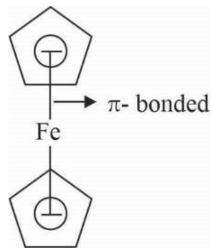
$\Rightarrow \quad -2.303RT \log Kc = \Delta H° - T\Delta S° = 77.2 \times 10^3 - 400 \times 122 = 28400J$

So $\log Kc = \left(\frac{-28400}{2.303 \times 8.314 \times 400}\right) \qquad \therefore \qquad [Kc = 1.958 \times 10^{-4}]$

12.(C) σ bonded organometallic compound $\Rightarrow$ $M-C$
$\sigma-$bond

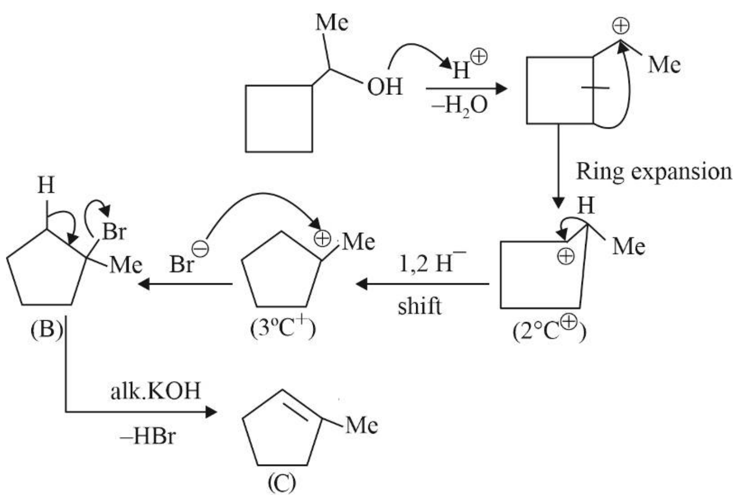
and in $\pi-$ bonded organo metallic compound
$M-C$
π bond

In ferrocene, there is π- bond

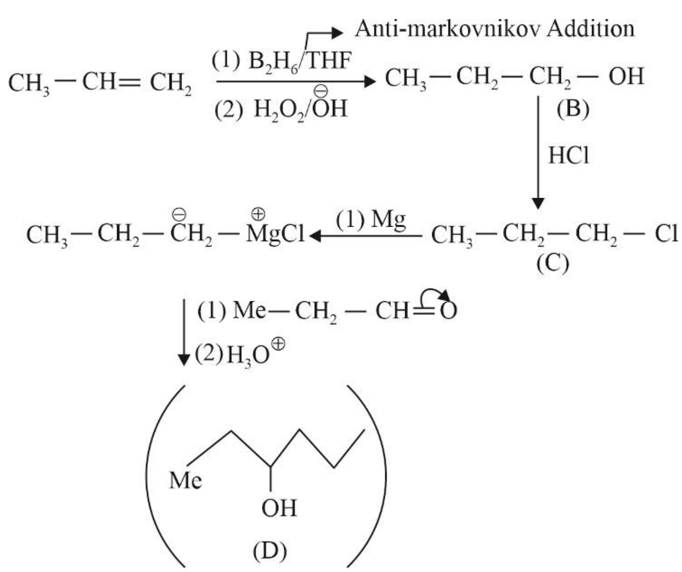


13.(D) Factual:
Lanthanoid $\Rightarrow$ 57 to 71 and actinoids $\Rightarrow$ 89 to 103

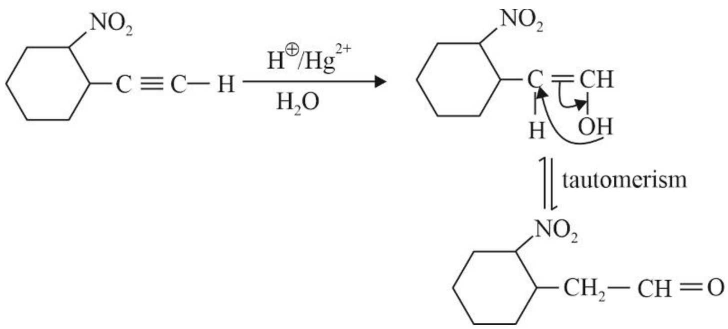
14.(C)



15.(C)



16.(C)



17.(B) $Sm \Rightarrow +2,+3\left(4f^6,6s^2\right)$; $Gd \Rightarrow +3\left(4f^5 5d^1 6s^2\right)$

$Tm \Rightarrow +2+3\left(4f^{13},6s^2\right)$; $Nd \Rightarrow +2,+3,+4\left(4f^4,6s^2\right)$

18.(B) Factual

(A) Magnalium contain Al and Mg **(B)** Bell metal $\Rightarrow$ Cu and Sn

(C) Gun metal $\Rightarrow$ Cu, Sn and Zn **(D)** Chrome steel $\Rightarrow$ Fe and Cr.

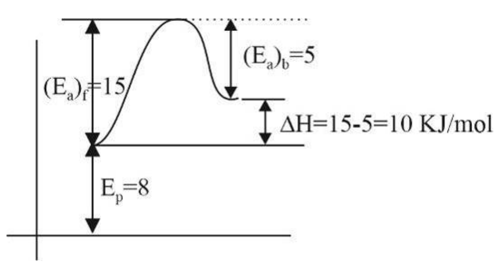
19.(C) Factual on lyophilic colloids

(A) It is easy to prepare **(B)** Stable

(C) Revisable

(D) Viscosity is high and surface tension is low for DP. and DM.

20.(A)



SECTION-2

21.(19.23) $\because$ % of S in the compound

$$= \frac{32}{233} \times \frac{\text{mass of } BaSO_4}{\text{mass of compound}} \times 100 = \frac{32 \times 0.35 \times 100}{233 \times 0.25} = 19.227 \simeq 19.23$$

22.(483.60) From $\Delta T_f = K_f \times m$, and $\Delta T_f = 15°C$

$$\therefore \quad m = \frac{\Delta T_f}{K_f} = \frac{15}{1.86} = 8.06$$

So the amount of propyl alcohol to be added.

$= m \times \text{mol wt} = 8.06 \times 60 = 483.6g$

23.(7.65) Mol wt of propane $= 44g$ and weight of propane $= 12.0g$

So, using $PV = nRT$

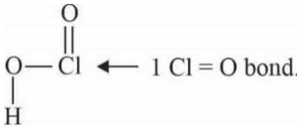
$$V = \left(\frac{nRT}{P}\right) = \frac{\left(\frac{12}{44}\right) \times 0.082 \times 333}{(740/760)} = 7.648L$$

24.(0.82) Initial m equivalent of $Cu^{2+} = 200 \times 0.5 \times 2 = 200$ m eq

So electricity passed $= \frac{0.965 \times 3600}{96500} = 36 \times 10^{-3}$ eq. $= 36$ m eq

m eq of $CuBr_2$ remaining $= 200 - 36 = 164$ $\therefore$ $N = \frac{meq}{V(\text{in ml})} = \left(\frac{164}{200}\right) = 0.82$

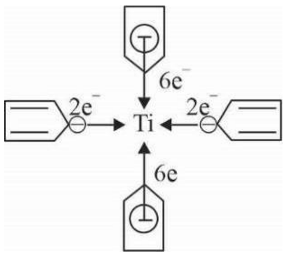
25.(1) Chlorous acid $\Rightarrow HClO_2$



26.(4) Factual

$\Rightarrow$ leaching methods is used for those metal in which metal is more soluble than impurities and these are Al, Au, Ag, low grade Cu

27.(34)

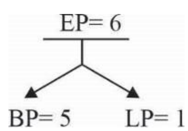


Total number of electron in $Ti = 22$

Total number of electron in $Ti^{4+} = 22 - 4 = 18$ So EAN value of $Ti = 18 + 12 + 4 = 34$

28.(3) According to VSEPR theory :

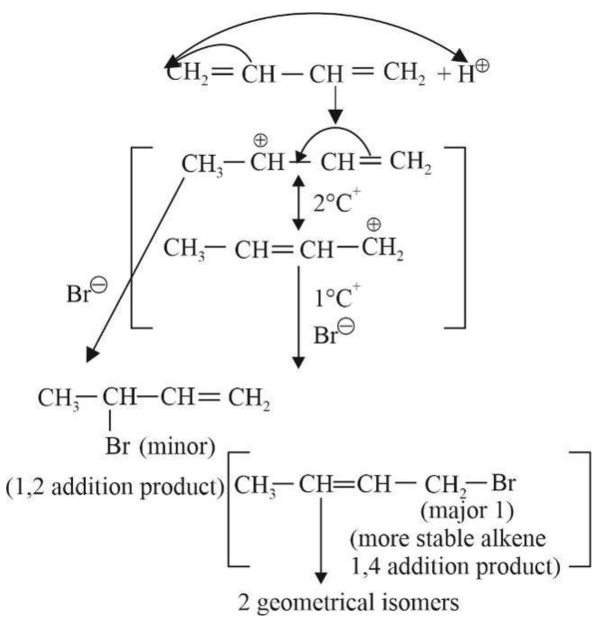
For the species which contain



Has square pyramidal shape

These are $BrF_5, ClF_5, XeOF_4$

29.(2)



30.(3) Key point: Fehling solution do not oxidise aromatic aldehydes, except when EWG present at O / P position

So (I), (III), (IV) not show this test.

MATHEMATICS

SECTION-1

1.(C) Apply $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 0 & \sin\theta - 3\cos\theta & 0 \end{vmatrix}$$

$\Delta = (3\cos\theta - \sin\theta)^2 \Rightarrow \Delta_{\max} = 10$

2.(B)

p	q	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T

3.(C) A: 3 numbers are in A.P.

B: 3 numbers are even

$$P(B/A) = \frac{n(A \cap B)}{n(A)} = \frac{{}^{25}C_2 + {}^{25}C_2}{{}^{50}C_2 + {}^{50}C_2} = \frac{25 \times 24}{50 \times 49} = \frac{12}{49}$$

4.(C) Solving, $x^2 - 9 = kx^2 \Rightarrow x^2(k-1) + 9 = 0 \Rightarrow x_1 + x_2 = 0$ and $x_1 x_2 = \frac{9}{k-1}$

$|x_1 - x_2| = 10 = \sqrt{(x_1 + x_2)^2 - 4x_1x_2} \Rightarrow \quad k = \frac{16}{25}$

5.(C) $\beta^2 - 6\beta + 12 = 0 \qquad \beta - 6 = \frac{-12}{\beta}$

$\therefore$ Reqd. Expression is $\left((\alpha-2)^3\right)^4 + \frac{(-12)^{12}}{(\alpha\beta)^{12}} - 1$

$$= \left(\alpha^3 - 8 - 6\alpha(\alpha-2)\right)^4 + \frac{(12)^{12}}{(12)^{12}} - 1 = \left(\alpha\left(\alpha^2 - 6\alpha + 12\right) - 8\right)^4 = 8^4 = 2^{12}$$

6.(D) $\log_6(abc) = 6 \Rightarrow abc = 6^6$

$a = \frac{b}{r}$ & $c = br$

$\Rightarrow \quad b = 36$ and $a = \frac{36}{r}$

$\Rightarrow \quad r = 2, 3, 4, 6, 9, 12, 18$

Also $b - a = 36\left(1 - \frac{1}{r}\right)$ is a perfect cube.

$\therefore \quad r = 4 \Rightarrow \quad a + b + c = 9 + 36 + 144 = 189$

7.(B) Parallel vector $\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 4 \\ 3 & -2 & 1 \end{vmatrix} = 5\hat{i} + 8\hat{j} + \hat{k}.$

Normal vector of plane $\vec{n} = 2\hat{i} - \hat{j} + a\hat{k}$

For the given condition $\vec{b}.\vec{n} = 0 \quad \Rightarrow 10 - 8 + a = 0 \quad \Rightarrow \quad a = -2$

8.(C) $f'(x)=\cos x+\sin x-k\le 0\,\forall x\in R$

$k\ge\sqrt{2}$

9.(B) Equation of tangent is $\frac{x}{a}\left(\frac{1}{2}\right)+\frac{y}{b}\left(\frac{\sqrt{3}}{2}\right)=1$ …(i)

Equation of auxiliary circle is $x^2+y^2=a^2$ …(ii)

Homogenising (ii) with (i) and making coefficient of x^2 + coefficient of $y^2=0$

$\Rightarrow \quad \frac{3a^2}{4b^2}=\frac{7}{4}\Rightarrow e=\frac{2}{\sqrt{7}}$

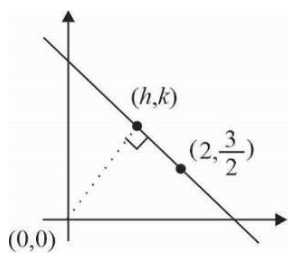
10.(D) $C.V.=\frac{\sigma}{\bar{x}}\times 100\Rightarrow \sigma=\frac{C.V.\times\bar{x}}{100}$

$\therefore \quad \sigma_1=\frac{50\times 30}{100}=15$ and $\sigma_2=\frac{60\times 25}{100}=15 \Rightarrow \quad \sigma_1-\sigma_2=0$

11.(C) $\left(5^3\right)^{740}5^2=5^2\left(125\right)^{740}=5^2\left(126-1\right)^{740}$

So remainder $=4$.

12.(C) $\frac{k-\frac{3}{2}}{h-2}\times\frac{k-0}{h-0}=-1$



$\Rightarrow \quad k\left(2k-3\right)=-2\left(h-2\right)h$

$\Rightarrow \quad 2h^2+2k^2-4h-3k=0$

$2x^2+2y^2-4x-3y=0$

13.(B) $\frac{6!}{5!1!}\times 2!+\frac{6!}{4!2!}\times 2!+\frac{6!}{\left(3!\right)^2 2!}\times 2!=62$

14.(C) $\frac{dy}{dx}=\left(e^y-x\right)^{-1}$

$\frac{dx}{dy}=e^y-x$

$\frac{dx}{dy}+x=e^y\Rightarrow$ I.F. $=e^{\int dy}=e^y$

$\therefore \quad xe^y=\int e^y e^y dy+c$

$xe^y=\frac{e^{2y}}{2}+c$

$y\left(0\right)=0 \quad \Rightarrow \quad c=-\frac{1}{2}$

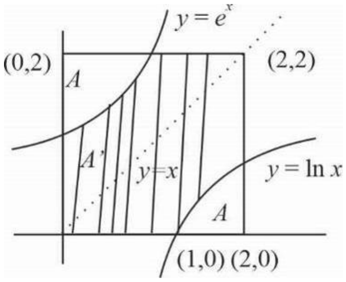
$\therefore \quad x=\frac{e^y}{2}-\frac{1}{2}e^{-y} \quad \Rightarrow \quad e^y-e^{-y}=2x$

$e^{2y}-2xe^y-1=0$

$e^y=x\pm\sqrt{x^2+1} \quad \Rightarrow \quad e^y=x+\sqrt{x^2+1}$

$y=\ln\left(x+\sqrt{x^2+1}\right)$

15.(A)



$$A=\int_1^2 \ln x\,dx = 2\ln 2-1$$

$$A' = 4-2(2\ln 2-1) = 6-4\ln 2$$

16.(C) $\alpha+\beta+\gamma=-a,\quad \alpha\beta+\beta\gamma+\gamma\alpha=b,\ \alpha\beta\gamma=-c$

$$\begin{vmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{vmatrix}=0 \Rightarrow \alpha^3+\beta^3+\gamma^3=3\alpha\beta\gamma \Rightarrow \alpha=\beta=\gamma \Rightarrow \frac{a^2}{b}=\frac{9\alpha^2}{3\alpha^2}=3$$

17.(C) $I_1=\int_0^1 \underbrace{\left(1-x^4\right)^7}_{I}.\underbrace{1}_{II}\,dx$

$$=\left(1-x^4\right)^7.x\Big|_0^1-\int_0^1 7\left(1-x^4\right)^6\left(-4x^3\right)x dx=-28\int_0^1\left(1-x^4\right)^6\left(1-x^4-1\right)dx$$

$$I_1=-28\int_0^1\left(1-x^4\right)^7+28\int_0^1\left(1-x^4\right)^6 dx$$

$$I_1=-28I_1+28I_2$$

$$29I_1=28I_2$$

$$\frac{I_1}{I_2}=\frac{28}{29} \Rightarrow \frac{29}{4}\frac{I_1}{I_2}=\frac{29}{4}\times\frac{28}{29}=7$$

18.(B) Let $S=z+2z^2+3z^3+------+18z^{18}$

$zS = \qquad \underline{z}^2+2z^3+-------+18z^{19}$

$-$

$$(1-z)S=z+z^2+z^3+-------+z^{18}-18z^{19}$$

$$(1-z)S=\frac{z\left(z^{18}-1\right)}{z-1}-18z^{19}$$

Now $z=\cos 20^\circ+i\sin 20^\circ \Rightarrow z^{18}=1$

Also $|z|=1$

$\Rightarrow \qquad (1-z)S=-18z$

$\Rightarrow \qquad S=\dfrac{-18z}{1-z}$

$$|s|^{-1}=\left|\frac{1-z}{-18z}\right|=\frac{|1-z|}{18}$$

$$=\frac{1}{18}\left|1-\cos 20^\circ-i\sin 20^\circ\right| = \frac{1}{18}\left|2\sin^2 10^\circ-2i\sin 10^\circ\cos 10^\circ\right|=\frac{1}{18}2\sin 10^\circ\left|\sin 10^\circ-i\cos 10^\circ\right|=\frac{1}{9}\sin 10^\circ$$

19.(A) $\lim_{x\to\infty}\frac{\int_0^x \tan^{-1} t\,dt}{\sqrt{x^2+1}}=\lim_{x\to\infty}\frac{\tan^{-1}x}{\frac{1}{2\sqrt{x^2+1}}\times 2x}=\lim_{x\to\infty}\left(\tan^{-1}x\right)\frac{\sqrt{x^2+1}}{x}=\frac{\pi}{2}$

20.(C) a^2, b^2, c^2.............$A.P.$

$a^2+1,\ b^2+1,\ c^2+1$..............$A.P.$

$a^2+ab+bc+ca,\ b^2+ab+bc+ca,\ c^2+ab+bc+ca$.......$A.P.$

$\Rightarrow\quad (a+b)(a+c),(a+b)(b+c),(b+c)(c+a)$.......$A.P.$

$\frac{1}{b+c},\ \frac{1}{c+a},\frac{1}{a+b}$.......$A.P.$

$\Rightarrow\quad b+c,\ c+a,\ a+b$...........$H.P.$

SECTION-2

21.(324) Volume $=\left|\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b\end{vmatrix}\right|=\left|3abc-a^3-b^3-c^3\right|$

$=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)=(a+b+c)\left((a+b+c)^2-3(ab+bc+ca)\right)$

$=9(81-3\times 15)=324$

22.(16) $A=\{1,2,3,4,5\}\quad B=\{0,1,2,3,4\}$

No. of elements common in $A\ \&\ B=4$.

$\therefore$ No. of elements common in $A\times B$ and $B\times A=4\times 4=16$

23.(4) $x^2+y^2-6x+8y+24=0$ is circle having centre $(3,-4)$ & $r=\sqrt{9+16-24}=1$

$\sqrt{x^2+y^2}$ min. is min. distance from origin $=4$

$\therefore$ minimum value of $\log_2\left(x^2+y^2\right)=\log_2 16=4$

24.(4) RHL & LHL $\quad\lim_{x\to 0}\left(\sin\frac{2x^2}{a}+\cos\frac{3x}{b}\right)^{\frac{ab}{x^2}}$

$=e^{\lim_{x\to 0}\left(\sin\frac{2x^2}{a}+\cos\frac{3x}{b}-1\right)\frac{ab}{x^2}}=e^{\frac{4b^2-9a}{2b}}$

$f(0)=e^3$ S

For continuity at $x=0$

Limit $=f(0)$

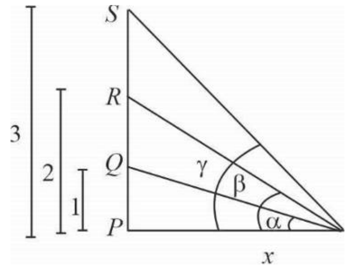
$\Rightarrow\ \frac{4b^2-9a}{2b}=3\quad\Rightarrow\quad 4b^2-6b-9a=0\ \ \forall b\in R$

$\Rightarrow\ D\ge 0\ \ \Rightarrow\ a\ge -\frac{1}{4}$

$a_{\min}=-\frac{1}{4}$

$\Rightarrow\quad \left|\frac{1}{a_{\min}}\right|=4.$

25.(1) $\tan\alpha = \frac{1}{x}$ $\quad \tan\beta = \frac{2}{x}$ $\quad \tan\gamma = \frac{3}{x}$



Now $\alpha+\beta+\gamma = 180°$

$\Rightarrow \quad \tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$

$\frac{1}{x}+\frac{2}{x}+\frac{3}{x} = \frac{1}{x}\frac{2}{x}\frac{3}{x}$

$\Rightarrow \quad x^2 = 1$

26.(10) $|x|^{3/5} = y$

$\Rightarrow y^2 - 26y - 27 = 0 \Rightarrow y = -1 \; or \; 27$

$\Rightarrow y = 27 \Rightarrow |x|^{3/5} = 3^3$

$|x| = 3^5 \Rightarrow \quad x = \pm 3^5$

Product of roots $= (3^5)(-3^5) = -3^{10}$

27.(0.60) A: runner succeeded exactly 3 times out of 5.

B: runner succeeds on the first trial.

$$P(B/A) = \frac{P(B\cap A)}{P(A)} \quad = \frac{p\left({}^4C_2 p^2(1-p)^2\right)}{{}^5C_3 p^3(1-p)^2} = \frac{3}{5} = 0.6$$

28.(1) $\tan x + \frac{\tan x + \sqrt{3}}{1-\sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1+\sqrt{3}\tan x} = 3$

$\tan x + \frac{8\tan x}{1-3\tan^2 x} = 3$

$\frac{\tan x(1-3\tan^2 x) + 8\tan x}{1-3\tan^2 x} = 3$

$\frac{3(3\tan x - \tan^3 x)}{1-3\tan^2 x} = 3$

$\Rightarrow \; 3\tan 3x = 3$

$\tan 3x = 1$

29.(360) ${}^6P_4 = \frac{6!}{2!} = 360$

30.(0) Put $x^{3/2} = t$

$\sqrt{x}dx = \frac{2}{3}dt$

$\Rightarrow \; \frac{2}{3}\int\frac{dt}{\sqrt{1-t^2}} = \frac{2}{3}\sin^{-1}t + c$

$= \frac{2}{3}\sin^{-1}x^{3/2} + c$

$\Rightarrow \; g(x) = \sin^{-1}x$

$g(0) = 0$